

Markscheme

November 2015

Mathematical Studies

Standard level

Paper 1





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Paper 1 Markscheme Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

The number of marks for each question is 6.

1 Abbreviations

The markscheme may make use of the following abbreviations:

- M Marks awarded for Method
- A Marks awarded for an Answer or for Accuracy
- **C** Marks awarded for **Correct** answers (irrespective of working shown)
- R Marks awarded for clear Reasoning
- ft Marks that can be awarded as **follow through** from previous results in the question

2 Method of Marking

- (a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If the candidate has full marks on a question use the *C6* annotation, if the candidate has made an attempt but scores zero marks use *C0*. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations **MUST** be shown.
- (c) In this paper, if the **correct answer is seen on the answer line the** maximum mark is awarded. **There is no need to check the working!** Award *C* marks and move on.
- (d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.
- (e) If the **answer is wrong**, marks should be awarded for the working according to the markscheme.
- (f) Working crossed out by the candidate should not be awarded any marks. Where candidates have written two solutions to a question, only the first solution should be marked.
- (g) A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.
- (h) If correct working results in a correct answer **in the working box** but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer **as long as it is unambiguous**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

Example:

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685 (incorrect decimal value)	Award the final (A1) (ignore the further working)
2.	(x-6)(x+1)	x = 6 and -1	Do not award the final (A1) (see next example)

Example: Factorise $x^2 - 5x - 6$

Marksch	neme	Candidates' Scripts	Marking
(x-6)(x+1)	(A1)(A1)	(i) Answer line: $(x+6)(x+1)$	(A0)(A1)
		(ii) Working box: $(x-6)(x+1)$ followed by $x=6$ and -1 , or just $6,-1$	(A1)
		in either working box or on answer line.	(A0)

3 Follow through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **(ft)**.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, there must be working present for that part. An isolated follow through answer, without working is regarded as incorrect and receives no marks even if it is approximately correct.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

	Markscheme	Candidates' Scripts		Marking
(a)	$\frac{\sin A}{3} = \frac{\sin 30}{4} \qquad (M1)(A1)$	(a)	$\frac{\sin A}{4} = \frac{\sin 30}{3}$	(M1)(A0)
				(use of sine rule but with wrong values)
	$A = 22.0^{\circ} (22.0243)$ (A1)		, ,	(A0) here was not marked (ft) and because there was an earlier uestion part.)
(b)	$x = 7 \tan (22.0243^{\circ})$ (M1) = 2.83 (2.83163) (A1)(ft)	(b)	case (i) $x = 7 \tan 41.8^{\circ}$ = 6.26 case (ii) 6.26	(M1) (A1)(ft) (C0) since no working shown

4 Using the Markscheme

(a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.

The only exception will be for an answer where the accuracy is specified in the question – see section 5.

- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** is not awarded for a correct answer if no reason or the wrong reason is given.
- (c) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' *etc*.

$$\sin \theta$$

(d) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\cos \theta$ for $\tan \theta$. On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value
$$\left(\text{for example }\frac{2}{3}\text{ if applicable}\right);$$

the full calculator display in the form 2.83163... as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

(e) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1.7; 1.7; 1,7.

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49.

Different descriptions of an interval: 3 < x < 5; (3, 5); 3 < 5.

Different forms of notation for set properties (e.g. complement): A'; \overline{A} ; A^c ; U - A; $(A; U \setminus A)$.

Different forms of logic notation:
$$\neg p$$
; p' ; \bar{p} ; \bar{p} ; $\sim p$.
$$p \Rightarrow q$$
; $p \rightarrow q$; $q \leftarrow p$.

Significance level may be written as α .

(f) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader. As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award (A1) and ignore subsequent rounding.

Note: The unrounded answer may appear in either the working box or on the final answer line.

2. If the candidate's unrounded answer is **not** seen then award **(A1)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.

Note: If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples which follow.

	If candidates final answer is given							
	Exact or to 4 or more sf (and would round to the correct 3 sf)	Correct to 3 sf	Incorrect to 3 sf	Correct to 2 sf ³	Incorrect to 2 sf	Correct or incorrect to 1		
Unrounded answer seen ¹	Award the	Award the final (A1) irrespective of correct or incorrect rounding						
Unrounded answer not seen ²	(A1)	(A0)	(A1)	(A0)	(A0)			
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen.3						

Examples:

Markschem	е		Candidates' Scripts	Marking
9.43 (9.43398)	(A1)	(i)	9.43398 is seen in the working borfollowed by 9; 9.4; 9.43; 9.434 etc (correctly rounded)	
		(ii)	9.43398 is seen in the working bor followed by 9.433; 9.44 <i>etc</i> . (incorrectly rounded)	
		(iii)	9.4	(A1)
		(iv)	9	(A0) (correct to 1 sf)
		(v)	9.3 (inc	(A0) correctly rounded to 2 sf)
		(vi)	9.44 (inc	(A0) correctly rounded to 3 sf)

Markschem	е		Candidates' Scripts	Marking
7.44 (7.43798)	(A1)	(i)	7.43798 is seen in the working box followed by 7; 7.4; 7.44; 7.438 etc. (correctly rounded)	(A1)
		(ii)	7.43798 is seen in the working box followed by 7.437; 7.43 <i>etc.</i> (incorrectly rounded)	
		(iii)	7.4	(A1)
		(iv)	7	(A0) (correct to 1 sf)
		(v)	7.5 (inco	(A0) rrectly rounded to 2 sf)
		(vi)	7.43 <i>(inco</i>	(A0) rrectly rounded to 3 sf)

Example: ABC is a right angled triangle with angle $ABC = 90^{\circ}$, AC = 32 cm and AB = 30 cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) BC = $\sqrt{32^2 - 30^2}$ (M1) Award (M1) for correct substitution in Pythagoras' formula $= 11.1 \left(\sqrt{124}, 11.1355 \right) \text{(cm)} \text{ (A1)}$	(a) BC = $\sqrt{32^2 - 30^2}$ 11 (cm) (2 sf answer on	(M1) (A1) ly seen, but correct)
(b) Area = $\frac{1}{2} \times 30 \times 11.1355$ (M1) Award (M1) for correct substitution in area of triangle formula	(b) case (i) $Area = \frac{1}{2} \times 30 \times 11$	(M1) (working shown)
= $167(167.032)$ (cm ²) (A1)(ft)	$=165 (cm^2)$	<i>(A1)</i> (ft)
	case (ii) =165 (cm²) (No working show treated as a ft, so no m	(M0)(A0)(ft) vn, the answer 11 is varks awarded here)

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (e.g. $\sqrt{13}$, $2^{\frac{2}{3}}$, $\sqrt[4]{5}$,) may be accepted as exact answers. All other powers (eg, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (ft) mark regardless of an immediately preceding (MO).

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme			Candidates' Scripts	Marking
Chi-squared		(a)	7.7	(A2)
7.68 (7.67543)	(A2)	(b)	7.67	(A1)
		(c)	7.6	(A1)
		(d)	8	(A0)
		(e)	7	(A0)
		(e)	7.66	(A0)

Regression line

Markscheme		Candidates' Scripts	Marking
y = 0.888x + 13.5 (A2) (y = 0.887686x + 13.4895)	(a)	y = 0.89x + 13	(A2) (both accepted)
If an answer is not in the form of an equation award at most $(A1)(A0)$.	(b)	y = 0.88x + 13	(A1) (one rounding error)
	(c)	y = 0.88x + 14	(A1) (rounding error repeated)
	(d)	(i) $y = 0.9x + 13$	
		(ii) $y = 0.8x + 13$	(A1) (1 sf not accepted)
	(e)	0.88x + 14 (two round	(A0) ling errors and not an equation)

Maximum/minimum/points of intersection

Markscheme		Candidates' Scripts	Marking
(2.06, 4.49) (A1)(A1) (2.06020, 4.49253)	(a)	(2.1, 4.5)	(A1)(A1) (both accepted)
	(b)	(2.0, 4.4)	(A1) (same rounding error twice)
	(c)	(2.06, 4.4)	(A1) (one rounding error)
	(d)	(2, 4.4)	(A0) sf not accepted, one rounding error)

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2 dp.

Markscheme				Marking	
\$231.62 (231.6189)	(A1)	(i)	231.6		(A0)
		(ii)	232	(Correct roundi	(A0) ng to incorrect level)
		(iii)	231.61		(A0)
		(iv)	232.00	(Parts incorrect round	(A0) (iii) and (iv) are both ding to correct level)

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two marks per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded (A1) under the accuracy rules given in Section 5.

	Markschem	ie		Can	Marking	
(a)	37000m^2	(A1)	(a)	36000 m^2	(Incorrect answer so u	(A0) inits not considered)
(b)	$3200\mathrm{m}^3$	(A1)	(b)	$3200 \mathrm{m}^2$		(A0) (Incorrect units)

If no method is shown and the answer is correct but with incorrect or missing units award the C marks with a one mark penalty.

8 Graphic Display Calculators

Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment "I used my GDC" cannot receive a method mark.

1.

	N	\mathbb{Z}	Q	\mathbb{R}
$\sqrt{14}$				✓
sin 30°			✓	✓
4	✓	✓	✓	✓
-3		✓	✓	✓
4.12×10¹			✓	✓
$3\frac{1}{3}$			√	√

(A6)(C6)

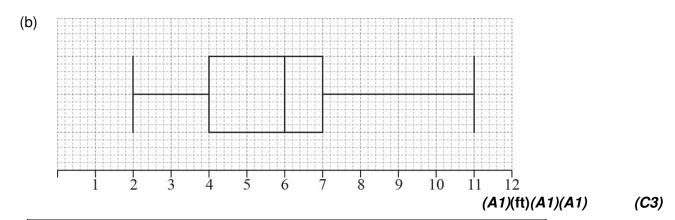
Note: Award (A1) for each completely correct row, (A0) otherwise. Allow X symbols in the blank spaces.

[6 marks]

2. (a) (i) 6 (M1)(A1)

> **Note:** Award *(M1)* for an ordered list of numbers seen. Award (A2) for 6.

(ii) 7 (A1)(C3)



Note: Follow through from part (a).

Award (A1)(ft) for their median,

(A1) for quartiles correct,

(A1) for endpoints correct.

If a horizontal line goes through the box, award at most (A1)(A1)(A0).

Award at most (A0)(A1)(A1) if a ruler has not been used.

3. (a) 5000×1.1824 (M1) 5912 (BRL) (A1) (C2)

(b)
$$\left| \frac{6000 - 5912}{5912} \right| \times 100$$
 (M1)

Note: Award *(M1)* for their correctly substituted percentage error formula. Accept a method in two steps where " $\times 100$ " is implicit from their answer.

Note: Follow through from part (a). Accept 1.49%. Do not accept 0.0149. Do not accept a negative answer.

(c)
$$\frac{\text{their part (a)} - 4000}{1.1824}$$
 (M1)

Note: Award (M1) for subtracting 4000 from their part (a) and dividing by 1.1824.

Note: Answer must be given to 2 decimal places for the final *(A1)* to be awarded. Follow through from part (a).

4. (a)
$$28 = u_1 + 6d$$
; $37 = u_1 + 9d$ **OR** $\frac{37 - 28}{3}$ (*M1*)

Note: For the first method, award *(M1)* for two correctly substituted arithmetic sequence formulae.

$$d = 3$$
 (A1)

(b)
$$28 = u_1 + 6(3)$$
 OR $37 = u_1 + 9(3)$ **(M1)**

Note: Award *(M1)* for correctly substituting their part (a) into substituted arithmetic sequence formula.

$$u_1 = 10$$
 (A1)(ft) (C2)

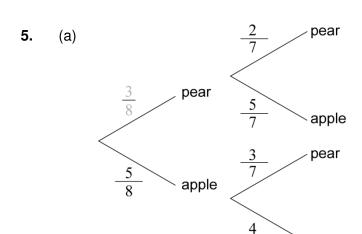
Note: Follow through from part (a).

(c)
$$S_{200} = \frac{200}{2} (2(10) + (200 - 1)3)$$
 (M1)

Note: Award *(M1)* for correctly substituting their part (a) and part (b) into arithmetic series formula.

$$=61700$$
 (A1)(ft) (C2)

Note: Follow through from parts (a) and (b).



(A1)(A1)(A1) (C3)

Note: Award *(A1)* for each correct pair of branches.

Accept decimal or percentage responses as equivalent forms on branches.

apple

(b)
$$\left(\frac{3}{8} \times \frac{2}{7}\right) + \left(\frac{5}{8} \times \frac{4}{7}\right)$$
 (A1)(ft)(M1)

Note: Award *(A1)*(ft) for two products consistent with their tree diagram, *(M1)* for addition of their products.

$$\frac{26}{56} \left(\frac{13}{28}, 0.464285..., 46.4\% \right)$$
 (A1)(ft) (C3)

Note: Follow through from their tree diagram, provided all the probabilities are between zero and 1.

[6 marks]

(b) the preferred drink is independent of gender (A1) (C1)

Note: Accept there is no association between preferred drink and gender. Do not accept "not related" or "not correlated" or "influenced".

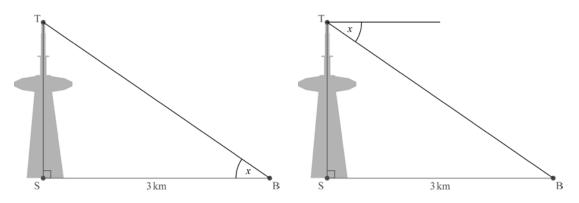
(d) reject
$$H_0$$
 (A1)(ft)
 $11.0 > 5.99$ OR $0.00403600... < 0.05$ (R1) (C2)

Note: Follow through from part (c). Do not award (A1)(ft)(R0).

Reference to the null hypothesis must be explicit for the (A1)(ft) to be awarded; do not accept "It is rejected" or similar.

A comparison of numbers must be seen to be awarded the reasoning mark. Award *(R1)* for comparison of χ values consistent with their answer to part (c) OR for correct comparison of p-values but only if p-value (0.004036...) is seen.

- 7. (a) $0.328 \, (km)$ (C1)
 - (b) a line on TB drawn and angle x correctly marked on the diagram (A1)



(c) $\tan x = \frac{0.328}{3}$ (M1)

Note: Award *(M1)* for **correct** substitution into the angle formula. Award *(M1)* if Pythagoras' theorem is used correctly ie $BT^2 = 0.328^2 + 3^2$ and correct use of trig ratio.

$$x = 6.24(^{\circ})(6.23955...^{\circ})$$
 (A1)(ft) (C2)

Note: Follow through from part (a) and their angle marked in part (b). Award *(A1)(A0)* for 0.109 (0.108900...) irrespective of whether there is working.

(d)
$$d = \sqrt{0.328^2 + 3^2}$$
 (M1)

Note: Award (M1) for their correct substitution into Pythagoras' theorem.

$$d = 3.02 (3.01787...) (km)$$
 (A1)(ft) (C2)

Note: Follow through from part (a). Accept alternative methods.

8. (a) $\frac{99921-97579}{97579}$ (×100)

Note: Award *(M1)* for subtraction **and** division by 97 579.

2.40 % (2.40010...) (A1) (C2)

(b) $97579 \times (1.024)^8$ **OR** $99921 \times (1.024)^7$ (*M1*)

Note: Award *(M1)* for their correct expression.

118000 (117966...) (A1)(ft) (C2)

Note: Follow through from part (a).

(c) 1.18×10^5 $(1.17966... \times 10^5)$ (A1)(ft) (C2)

Note: Award *(A1)*(ft) for 1.18.

Award (A1)(ft) for $\times 10^5$.

Award (A0)(A0) for answers of the type: 118×10^3 .

Follow through from part (b) with the same level of accuracy.

9. (a)

p	q	$\neg p$	$\neg p \Rightarrow q$
Т	Т	F	Т
Т	F	F	Т
F	Т	Т	Т
F	F	Т	F

 $(A1) \qquad (C1)$

Note: Award (A1) for completely correct final column.

(b) if Emma does not play tennis, then Emma goes to the beach (A1)(A1) (C2)

Note: Award *(A1)* for "lf... then", *(A1)* for correct statements in the correct order. Accept "she" in place of the second "Emma".

(c) $p \Rightarrow \neg q$ (A1)(A1) (C2)

Note: Award *(A1)* for " \Rightarrow ", *(A1)* for correct symbols in the correct order. Accept $\neg q \Leftarrow p$.

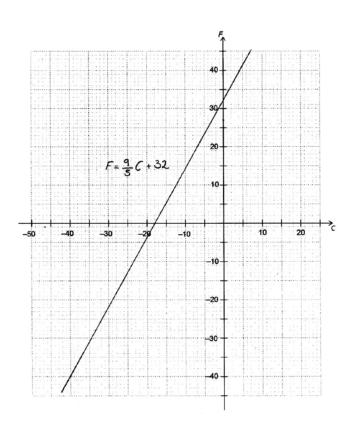
(d) neither (A1)(ft) (C1)

Note: Follow through from part (a).



 $(A1) \qquad (C1)$

(b)



(A1)(A1) (C2)

Note: Award *(A1)* for *y*-intercept at (0, 32), *(A1)* for correct gradient. Gradient can be checked by acceptable *x*-intercepts -18 < x < -17 or pass through (-40, -40) if *y*-intercept is correct. Award at most *(A1)(A0)* if not a straight line.

(c)
$$-10 \,(^{\circ}\text{C})$$
 (A1)(ft) (C1)

Note: Follow through from their graph in part (b).

(d) drawing C = F correctly onto the graph

(M1)

OR

drawing $C = \frac{5}{9}(F - 32)$ correctly onto the graph

(M1)

OR

$$x = \frac{9}{5}x + 32 (M1)$$

Note: Award *(M1)* for substituting x for y in the given equation.

$$(x =) -40$$
 (A1)(ft) (C2)

Note: Follow through from part (b) if a graphical method is used.

11. (a)
$$1200 \left(1 + \frac{3.4}{100 \times 4}\right)^{7 \times 4}$$
 (M1)(A1)

Note: Award *(M1)* for substitution into the compound interest formula. Award *(A1)* for correct substitutions.

OR

$$N = 7$$

 $I\% = 3.4$
 $PV = (\pm)1200$
 $P/Y = 1$
 $C/Y = 4$

(A1)(M1)

Note: Award (A1) for C/Y = 4 seen, (M1) for all other entries correct.

OR

$$N = 28$$

 $I\% = 3.4$
 $PV = (\pm)1200$
 $P/Y = 4$
 $C/Y = 4$

(A1)(M1)

Note: Award (A1) for C/Y = 4 seen, (M1) for all other entries correct.

$$(FV =) 1520.92$$
 (A1) (C3)

Note: The answer must be given to the correct two decimal places for the final *(A1)* to be awarded.

continued...

Question 11 continued

(b)
$$669 = 1200 \left(1 - \frac{r}{100} \right)^7$$
 (M1)(A1)

Note: Award *(M1)* for substitution into compound interest formula equated to 669 or equivalent. Accept a plus or minus symbol within the bracket. Award *(A1)* for correct substitutions.

OR

$$N = 7$$

$$PV = \pm 1200$$

$$FV = \mp 669$$

$$P/Y = 1$$

$$C/Y = 1$$

(A1)(M1)

Note: Award *(A1)* for $FV = \mp 669$ seen, *(M1)* for all other entries correct. PV and FV must have opposite signs.

$$r = 8.01 \, (\%) \, (8.00816...)$$
 (C3)

Note: Do not accept 8%. Award *(A2)* for an answer of -8.01 (%).

12. (a)
$$\frac{3}{4}$$
 (0.75) (C1)

Note: Follow through from part (a).

Note: Award (M1) for calculating subsequent terms until non-integer term reached.

$$(u_7 =) 364.5$$
 (A1)(ft) (C2)

Note: Accept "the 7th term" or "7" or u_7 as a final answer. Follow through from parts (a) and (b).

(d)
$$S_{20} = \frac{2048 \left(1 - \frac{3}{4}^{20}\right)}{\left(1 - \frac{3}{4}\right)}$$
 (M1)

Note: Award *(M1)* for correct substitution of their r and 2048 and 20 into geometric series formula i.e. $\frac{2048 \left(1 - (\text{their } r)^{20}\right)}{1 - \text{their } r}.$

Note: Follow through from part (a). If their $r = \frac{4}{3}$, the answer is unrealistic (as their total will be greater than $20 \times$ first term, which is impossible for a decreasing sequence). Award a maximum of *(M1)(A0)*.

13. (a)
$$2x + y = 60$$
 (A1)

Note: Accept (total) length = 2x + y. Answer must be an equation.

(b)
$$y = 60 - 2x$$

Note: Award *(M1)* for correctly rearranging their answer to part (a). Award *(M0)(A0)* for A = xy.

$$(A =) x(60-2x)$$
 OR $-2x^2+60x$ **(A1)(ft)**

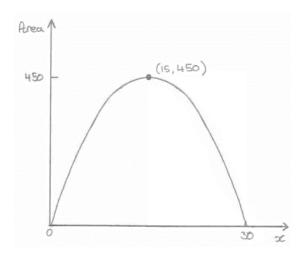
Note: Follow through from part (a).

(c)
$$0 = -4x + 60$$
 (M1)

Note: Award (M1) for correctly differentiating their answer to part (b) and equating to zero.

OR

a sketch of the graph of their answer to part (b) (M1)



Note: Award *(M1)* for sketch of parabola only in first quadrant starting at (0, 0).

$$x = 15$$
 (A1)(ft)

Note: Follow through from part (b). Award *(A0)* for x = 15 m seen without working; working might take the form of $\frac{-b}{2a} = \frac{-60}{-4}$.

$$450 \text{ (m}^2\text{)}$$
 (C3)

Note: Follow through from part (b).

14. (a) x = 1 (A1)(A1) (C2)

Note: Award **(A1)** for "x = (a constant)" and **(A1)** for "1".

(b) any one of the following four equations

(M1)

$$36a + 6b + 48 = 0$$

$$16a - 4b + 48 = 0$$

$$1 = -\frac{b}{2a}$$

b = 4

$$ax^2 + bx + 48 = k(x+4)(x-6)$$

$$a = -2$$

(A1)(ft)

(A1)(ft) (C3)

Note: Follow through from part (a). If a correct and b incorrect (or the other way round) then the answer is wrong; depending on the equations seen, award (M1)(A1)(A0) OR (M0)(A0)(A0), and if there is no working award (M0)(A0)(A0).

(c) y = 50 (A1)(ft) (C1)

Note: Follow through from parts (a) and (b), but only if the answer is greater than 48. The upper bound for acceptable ft values from the graph is 60.

[6 marks]

15. (a) 25 (A1) (C1)

(b) 200 (C1)

(c) $25 \times 8^t > 5000000$ (M1)(M1)

Notes: Award *(M1)* for left hand side, *(M1)* for right hand side. Accept an equality. Accept a list including at least N(5) and N(6). Award *(M1)* for N(5) and N(6) seen, *(M1)* for comparing N(5) and N(6) with $5\,000\,000$.

$$t > 5.86988...$$
 hours (A1)

$$t = 6 \text{ (hours)} \tag{C4}$$

Note: The final **(A1)** is awarded for correct rounding of *their* unrounded **non-integer** *t* value to the nearest integer. Award at most **(M0)(M1)(A1)(A1)** if *x* is used instead of *t*.